Seat No.: _____ Enrolment No._____

# GUJARAT TECHNOLOGICAL UNIVERSITY

**BE - SEMESTER–III(NEW) EXAMINATION – SUMMER 2023**

**Subject Code:2130002 Date:21-07-2023**

**Subject Name:Advance Engineering Mathematics**

**Time:02:30 PM TO 05:30 PM Total Marks:70**

**Instructions:**

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

**Q.1** **(a)** (i) Solve $3e^x \tan y\,dx + \left(1+e^x\right)\sec^2 y\,dy = 0$. **03**

(ii) Solve $\dfrac{dy}{dx} + \dfrac{4x}{x^2+1} = \dfrac{1}{\left(x^2+1\right)^3}$. **04**

**(b)** Find the Fourier series expansion of $f(x) = x^2 \;;\; -2 \le x \le 2$. **07**

**Q.2** **(a)** (i) Define Signum function and Triangular wave function. **03**

(ii) Solve $y'' + 4y = 8x^2$. **04**

**(b)** Find the power series solution of $\left(x^2+1\right)\dfrac{d^2y}{dx^2} + 2x\dfrac{dy}{dx} + xy = 0$. **07**

**OR**

**(b)** Find the Fourier series of $f(x) = \begin{cases} x^2 & ;\; x \in (0,\pi) \\ 0 & ;\; x \in (\pi, 2\pi) \end{cases}$ **07**

**Q.3** **(a)** (i) Find $L\left(t^2 \cos at\right)$. **03**

(ii) Find general solution of $y'' + 9y = \sec 3x$ by method of variation of parameters. **04**

**(b)** Using convolution theorem, find $L^{-1}\left\{\dfrac{1}{\left(s^2+a^2\right)^2}\right\}$. **07**

**OR**

**Q.3** **(a)** (i) Find $L^{-1}\left\{\dfrac{1}{\left(s+\sqrt{2}\right)\left(s-\sqrt{3}\right)}\right\}$. **03**

(ii) Solve $\left(D^2 - 2D + 1\right)y = x e^x \sin x$. **04**

**(b)** Solve the equation **07**
$y'' - 3y' + 2y = 4t + e^{3t}$, when $y(0) = 1$ and $y'(0) = -1$

**Q.4** **(a)** (i) Find $L\left\{\int_0^t e^{-u} \cos u\,du\right\}$. **03**

(ii) Find Fourier sine series of $f(x) = \pi - x, \quad 0 < x < \pi$. **04**

**(b)** Find the solution of $y'' + 4y = 2\sin 3x$ by the method of undetermined coefficients. **07**

**OR**

**Q.4** **(a)** (i) Find $L^{-1}\left[\log\left(\frac{s+a}{s+b}\right)\right]$. **03**

(ii) Solve $\frac{dy}{dx}+\frac{y}{x}=x^3y^3$. **04**

**(b)** Find the series solution using Frobenious method for $xy''+y'-y=0$. **07**

**Q.5** **(a)** (i) Derive partial differential equation by eliminating a and b from $z=(x-a)^2+(y-b)^2$. **03**

(ii) Solve $pz-qz=z^2+(x+y)^2$. **04**

**(b)** Solve $\left(D^2+DD'-6D'^2\right)z=y\cos x$. **07**

**OR**

**Q.5** **(a)** (i) Solve $p+q^2=1$. **03**

(ii) Using Charpit's method solve $z=pq$. **04**

**(b)** Solve $\frac{\partial u}{\partial x}+2\frac{\partial u}{\partial y}=0$ by separation of variable method. **07**

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