Enrolment No./Seat No_____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III (NEW) EXAMINATION – SUMMER 2024

Subject Code:2130002 Date:11-07-2024

Subject Name: Advance Engineering Mathematics

Time:10:30 AM TO 01:30 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			Marks
Q.1	**(a)**	Solve $9yy' + 4x = 0$.	**3**
	(b)	Solve the initial value problem $y' - (1 + 3x^{-1})y = x + 2; y(1) = e - 1$.	**4**
	(c)	Find the Fourier series of the function $f(x) = x^2; -\pi < x < \pi$.	**7**
Q.2	**(a)**	Find the general solution of $y'' + 3y' + 2y = 0$.	**3**
	(b)	Solve $y''' - 3y'' + 3y' - y = 4e^t$.	**4**
	(c)	Using the method of variation of parameters find the general solution of $(D^2 - 2D + 1)y = 3x^{\frac{3}{2}}e^x$.	**7**
		OR	
	(c)	Using the method of undermined coefficients, find a particular solution of $y'' - 4y' - 12y = 8x^2$.	**7**
Q.3	**(a)**	Obtain the Fourier series for the function $f(x)$ given by $f(x) = \begin{cases} 1 + \left(\frac{2x}{\pi}\right); -\pi \le x \le 0 \\ 1 - \left(\frac{2x}{\pi}\right); 0 \le x \le \pi \end{cases}$. Hence, deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$.	**7**
	(b)	A function $f(x)$ is defined by $f(x) = \begin{cases} 1; -1 \le x \le 1 \\ 0; otherwise \end{cases}$ Find the Fourier integral representation of $f(x)$. Hence, evaluate (a) $\int_0^\infty \frac{sin\lambda cosx\lambda}{\lambda} d\lambda$. (b) $\int_0^\infty \frac{sin\lambda}{\lambda} d\lambda$.	**7**
		OR	
	(a)	Find a cosine series of period 2π to represent $f(x) = sinx$ in $0 < x < \pi$. Also, graph the corresponding periodic continuation of $f(x)$. Hence deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots . = \frac{\pi}{4}$	**7**
	(b)	Determine the series solution for the differential equation $y'' + y = 0$ about $x_0 = 0$.	**7**
Q.4	**(a)**	Find the Laplace transform of $t^3 + e^{-3t} + t^{\frac{3}{2}}$.	**3**
	(b)	Find the Laplace transform of $cosh(kt)coskt$	**4**

(c) Find the inverse Laplace transform of $\frac{2s+3}{(s+2)(s+1)^2}$ **7**

OR

(a) Define (i) Gamma function and Beta function **3**
(ii) Write the relation between Beta and Gamma function.

(b) Find the Laplace transform of unit step function $f(t) = \begin{cases} 0; 0 \le t < k \\ 1; \quad t \ge k \end{cases}$ **4**

(c) Solve the IVP using the Laplace transform: **7**
$y'' + 4y = 0; y(0) = 1, y'(0) = 6.$

Q.5 **(a)** Form the partial differential equation by eliminating the arbitrary constants for $az + b = a^2x + y$. **3**

(b) Solve $(y + z)p - (x + z)q = x - y$. **4**

(c) Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$. **7**

OR

(a) Find the complete integral of $z = px + qy + pq$. **3**

(b) Solve $\left(D^2 + 10DD' + 25D'^2\right)z = e^{3x+2y}$. **4**

(c) The base of semi-infinite strip of metal plate is 30cm and is kept at 100°C. The two long edges are at zero temperature. Find the temperature at any point 15cm away from the base and situated midway between the long edges. **7**

****