

**GUJARAT TECHNOLOGICAL UNIVERSITY****BE- SEMESTER-III (NEW) EXAMINATION – WINTER 2024****Subject Code:2130002****Date:18-12-2024****Subject Name:Advance Engineering Mathematics****Time:10:30 AM TO 01:30 PM****Total Marks:70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

	Marks
<b>Q.1</b> (a) Solve: $x^2 dy + y(x+y)dx = 0$	<b>03</b>
(b) Find the Laplace transform of $t^2 \sin \pi t$	<b>04</b>
(c) Find a Fourier series for a periodic function $f(x)$ with period 2, where $f(x) = -1, \quad -1 < x < 0$ $= 1, \quad 0 < x < 1$	<b>07</b>
<b>Q.2</b> (a) Define Beta function, Gamma function and write the relation between Beta and Gamma function.	<b>03</b>
(b) Solve: $(x+1) \frac{dy}{dx} - y = e^{3x}(x+1)^2$	<b>04</b>
(c) Find the Fourier series of $f(x) = x +  x $ , $-\pi < x < \pi$ .	<b>07</b>
<b>OR</b>	
(c) Solve $y'' + 4y = 8x^2$ by using the method of undetermined coefficients.	<b>07</b>
<b>Q.3</b> (a) Laplace Find $L^{-1} \left\{ \frac{s^3 + 2s^2 + 2}{s^3(s^2 + 1)} \right\}$	<b>03</b>
(b) Find particular integral for an equation $(D^2 - 4D + 3)y = \sin 3x \cos 2x$	<b>04</b>
(c) Solve the IVP $y'' - 2y' = e^t \sin t$ , $y(0) = y'(0) = 0$	<b>07</b>
<b>OR</b>	
<b>Q.3</b> (a) By using first shifting theorem find $L\{(t+1)^2 e^t\}$	<b>03</b>
(b) Solve $\frac{d^2 y}{dx^2} + 4y = \tan 2x$ by using Variation of Parameter.	<b>04</b>
(c) Using convolution theorem find the inverse transform of $\frac{a}{s^2(s^2 + a^2)}$	<b>07</b>
<b>Q.4</b> (a) Solve $\frac{dy}{dx} + \frac{2y}{x} = \sin x$	<b>03</b>
(b) Find the Laplace Transform of $e^{-2t}(\sin 4t + t^2)$	<b>04</b>
(c) Find the Power series solution of the equation $(x^2 + 1)y'' + xy' - xy = 0$	<b>07</b>
<b>OR</b>	
<b>Q.4</b> (a) (I) Find the Laplace Transform of $\frac{\cos at - \cos bt}{t}$	<b>03</b>
(II) Find the Laplace transform of $L\{t^2 u(t-2)\}$	
(b) Find half-range cosine series for $f(x) = (x-1)^2$ , $0 < x < 1$	<b>04</b>
(c) Find the series solution of $xy'' + y' + xy = 0$	<b>07</b>

- Q.5** (a) (I) Find the complete integral of  $pq = 4z$  **03**  
 (II) Find the complete integral of  $p - x^2 = q + y^2$
- (b) Form a partial differential equation by eliminating arbitrary constants a and b from equation  $z = (x^2 + a)(y^2 + b)$  **04**
- (c) Using Charpti's method solve:  $z = pq$  **07**
- OR**
- Q.5** (a) Form a partial differential equation by eliminating arbitrary function from  $\phi(x^2 - y^2, xyz) = 0$  **03**
- (b) Solve  $x^2p + y^2q = z^2$  **04**
- (c) Solve the equation  $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ , given  $u(x, 0) = 4e^{-4x}$  by using Separation of variable method. **07**

\*\*\*\*\*