Enrolment No./Seat No_______________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER–I&II EXAMINATION – SUMMER 2025

Subject Code:3110014 **Date:24-06-2025**

Subject Name:Mathematics - 1

Time:10:30 AM TO 01:30 PM **Total Marks:70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			MARKS
Q.1	**(a)**	Evaluate: $\lim_{x\to 0}\left[x^{-2} - cosec^2\, x\right]$.	**03**
	(b)	Check the consistency of the system of linear equations. Solve it if consistent. $3x + y - 3z = 13,$ $2x - 3y + 7z = 5,$ $2x + 19y - 47z = 32.$	**04**
	(c)	Find the Fourier Series of the function $f(x) = x^2$ in the interval $(-\pi, \pi)$.	**07**
Q.2	**(a)**	Define the improper integrals of the first kind and the second kind. State the relation between Beta and Gamma function.	**03**
	(b)	Find the area of the surface of revolution generated by revolving the curve $x = y^3$ from $y = 0$ to $y = 2$.	**04**
	(c)	Find eigen values and corresponding eigen vectors of the matrix $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$.	**07**
		OR	
	(c)	Find the inverse of the matrix using Gauss-Jordan elimination method. $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{pmatrix}$.	**07**
Q.3	**(a)**	Determine whether $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$ exists and find it if exists.	**03**
	(b)	Find the equation of tangent plane and normal line to the surface $2xz^2 - 3xy - 4x = 7$ at the point $(1, -1, 2)$.	**04**
	(c)	Find the extreme values of the function $x^3 + y^3 - 63(x + y) + 12xy$.	**07**
		OR	
Q.3	**(a)**	State chain rule for $\frac{\partial u}{\partial x}$ for $u = f(v, w),\ where\ v = g(x, y), w = h(x, y)$. If $u = f(x - 2y, 2y - 3z, 3z - x)$ then show that $\frac{\partial u}{\partial x} + \frac{1}{2}\frac{\partial u}{\partial y} + \frac{1}{3}\frac{\partial u}{\partial y} = 0$.	**03**
	(b)	Find the directional derivative of $f(x, y, z) = xy^2 + yz^2$ at the point $(2, -1, 1)$ in the direction of the vector $\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$.	**04**

	(c)	The temperature $T(x,y,z)$ at any point in space is $T = 400xyz^2$. Find the highest temperature on surface of the sphere $x^2 + y^2 + z^2 = 1$.	**07**
Q.4	**(a)**	Evaluate $\int_{-1}^{1}\int_{0}^{2}(1-6x^2y)dx\,dy$.	**03**
	(b)	Find the value of $\iint_R (2x - y^2)\ dA$ over the triangular region R enclosed between the line $y = -x+1, y = x+1$ and $y = 3$.	**04**
	(c)	Change the order of integral and evaluate $\int_0^{4a}\int_{x^2/4a}^{2\sqrt{ax}} dy\,dx$, (a > 0).	**07**

OR

Q.4	**(a)**	Calculate $\int_0^2\int_1^z\int_0^{yz} xyz\ dx\,dy\,dz$.	**03**
	(b)	Evaluate $\int_0^1\int_x^1 \sin y^2\ \ dy\,dx$.	**04**
	(c)	Compute $\int_0^2\int_0^{\sqrt{2x-x^2}} \frac{x}{x^2+y^2}\ dy\,dx$ by transforming into polar coordinates.	**07**
Q.5	**(a)**	Define Monotonic sequence. Test the convergence of the sequence $\{2-(-1)^n\}$.	**03**
	(b)	Express the function $f(x) = \log(1+x)$ in power series using the formula of Maclaurin's series.	**04**
	(c)	(i) Test the convergence of $\sum_{n=1}^{\infty}\frac{n!}{(2n+1)!}$. (ii) State Cauchy's root test and discuss the convergence of $\sum_{n=1}^{\infty}\frac{1}{(\log n)^n}$.	**07**

OR

Q.5	**(a)**	State sandwich theorem for sequence. Show that the sequence $u_n = \frac{\sin n}{n}$ converges to zero.	**03**
	(b)	Using Taylor's theorem find the approximate value of $\sqrt{10}$ up to three decimal places.	**04**
	(c)	(i) Examine the convergence of the series $\frac{1}{1\cdot 2} - \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} - \frac{1}{5\cdot 6} + \frac{1}{7\cdot 8} - \cdots$ ... . (ii) State Cauchy's Integral test for convergence of series and test the convergence of the series $\sum_{n=1}^{\infty} n^2 e^{-n^3}$.	**07**

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