

Enrolment No./Seat No _____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER-I&II EXAMINATION – SUMMER 2025

Subject Code:3110015

Date:07-07-2025

Subject Name:Mathematics - 2

Time:10:30 AM TO 01:30 PM

Total Marks:70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

Q.1 (a) Show that **03**

$\vec{F} = (y^2 - z^2 + 3yz - 2x)\vec{i} + (3xz + 2xy)\vec{j} + (3xy - 2xz + 2z)\vec{k}$ is both solenoidal and irrotational.

(b) Find Fourier integral representation of function $f(x) = \begin{cases} 1 & |x| < 1 \\ 0 & |x| > 1 \end{cases}$ **04**

Hence, Evaluate $\int \frac{\sin \omega \cos \omega x}{\omega} d\omega$.

(c) Verify Green's theorem in the plane for $\int_C [(3x^2 - 8y^2)dx + (4y - 6xy)dy]$, where C is the boundary of the region bounded by $x = 0, y = 0, x + y = 1$. **07**

Q.2 (a) Solve $ye^x dx + (2y + e^x)dy = 0$ **03**

(b) Find the directional Derivative of $\phi = xy^2 + yz^2$ at the point $(2, -1, 1)$ in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$. **04**

(c) Find the power series solution of $\frac{d^2y}{dx^2} + xy = 0$. **07**

OR

(c) Find the series solution of $(1 - x^2)y'' - 2xy' + 2y = 0$ **07**

Q.3 (a) Solve $(4D^2 - 4D + 1)y = e^{\frac{x}{2}}$ **03**

(b) Solve $(D^2 + 6D + 8)y = \cos^2 x$ **04**

(c) Solve $(D^2 - 2D + 3)y = x^3 + \sin x$ by using the method of undetermined coefficients **07**

OR

Q.3 (a) Solve $(D^2 + a^2)y = \cos ax$ **03**

(b) Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = e^x \sin x + 7$ **04**

(c) Solve $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = \log x$. $\sin(\log x)$ by Cauchy's linear equation **07**

- Q.4 (a)** Find the Laplace Transform Of $f(t) = \sinh at$. **03**
- (b)** Find the Laplace Transform of $te^{4t}\cos 2t$. **04**
- (c)** Solve the differential equation using initial value problem $y'' + 6y' = 1, y(0) = 2, y'(0) = 0$. **07**
- OR**
- Q.4 (a)** Find the inverse Laplace transform of $\log \frac{(s^2+b^2)}{(s^2+a^2)}$ **03**
- (b)** Find the Laplace Transform of $\frac{e^{-2t}\sin 2t \cosh t}{t}$ **04**
- (c)** Find the Inverse Laplace Transform Of $\frac{5s+3}{(s-1)(s^2+2s+5)}$ **07**
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- Q.5 (a)** Solve $\frac{dy}{dx} + y \cot x = 2 \cos x$ **03**
- (b)** Solve $3x^4p^2 - xp - y = 0$ **04**
- (c)** Using the method variation of parameter solve the differential equation $(D^2 + 1)y = x \sin x$ **07**
- OR**
- Q.5 (a)** Solve $(xy - 2y^2)dx - (x^2 - 3xy)dy = 0$ **03**
- (b)** Solve $\frac{dy}{dx} + \frac{1}{x}y = \frac{y^2}{x^2}$ **04**
- (c)** Solve $(D^4 - 16)y = e^{2x} + x^4$ **07**

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