GUJARAT TECHNOLOGICAL UNIVERSITY
BE- SEMESTER–I&II EXAMINATION – SUMMER 2025

Subject Code:BE02000011   **Date:18-06-2025**

Subject Name:Mathematics – 2

Time:10:30 AM TO 01:30 PM   **Total Marks:70**

Instructions:
1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			Marks
Q.1	**(a)**	Prove that $f(z) = \frac{1}{z}$ is not differentiable at $z = 0$, by definition.	**03**
	(b)	Solve $\frac{dx}{dy} + \frac{x}{y} = x^3$.	**04**
	(c)	Check the whether the given matrix $A = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$ is diagonalizable? If so, then find matrix P such that $P^{-1}AP = D$.	**07**
Q.2	**(a)**	State Rouche's Theorem. Evaluate $\int_C e^z dz$ if C is the circle $|z| = 99$.	**03**
	(b)	Solve $y'' + 4y' + 4y = 0, y(0) = 1, y'(0) = 1$.	**04**
	(c)	(1) Evaluate $\int_C \frac{3z^2 + z}{z^2 - 1} dz$ where C is the circle $|z - 1| = 1$.	**03**
		(2) Define entire function. Prove that e^z is analytic at everywhere.	**04**
		OR	
	(c)	(1) Evaluate $\int_C Re(z)dz$ from $1 + i$ to $3 + 2i$ along the straight line $2y = x + 1$.	**03**
		(2) Find real part and imaginary part of the following functions (i) $\sin z$ (ii) $\ln z$.	**04**
Q.3	**(a)**	Solve $\frac{dy}{dx} + 2y \tan x = \sin x , y(0) = 2$.	**03**
	(b)	Solve the following linear system by Gauss elimination method: $x - 2y + 3z = -2, \quad -x + y - 2z = 3, \quad 2x - y + 3z = -7$.	**04**
	(c)	Define Harmonic function. Check whether $v = 3x^2 y - y^3$ is harmonic or not. If so, construct the analytic function $f(z) = u + iv$.	**07**
		OR	
Q.3	**(a)**	Test for exactness and solve $2xydx + x^2 dy = 0$.	**03**
	(b)	Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & -1 & 0 \\ -1 & 3 & 0 & -4 \\ 2 & 1 & 3 & -2 \\ 1 & 1 & 1 & -1 \end{bmatrix}$.	**04**
	(c)	Check whether $u = 3x - 2xy$ is harmonic or not. If so, find the harmonic conjugate v of u.	**07**

Q.4 **(a)** Solve $x^2 y\,dx - (x^3 + xy^2)dy = 0$. **03**

(b) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ using Gauss Jordan method. **04**

(c) Find all the Taylor's and Laurent series of $f(z) = \dfrac{-2z+3}{z^2-3z+2}$ with center 0. **07**

OR

Q.4 **(a)** Solve $xe^x(dx - dy) + e^x dx + ye^y dy = 0$. **03**

(b) Find inverse of the matrix $A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ by Cayley-Hamilton theorem. **04**

(c) State Cauchy Residue theorem. Evaluate $\int_C \dfrac{z^2}{(z-1)^2(z+2)}\,dz$ where C is the circle $|z| = 3$ using Cauchy Residue theorem. **07**

Q.5 **(a)** Solve $x^2 p^2 + 3xyp + 2y^2 = 0$. **03**

(b) Solve $xy'' + y' = \dfrac{12\log x}{x}$. **04**

(c) Solve $y'' - 2y' + y = xe^x \sin x$ by using the method of variation of parameter. **07**

OR

Q.5 **(a)** Solve $\left(x\dfrac{dy}{dx} - y\right)\left(1 - \dfrac{dy}{dx}\right) = \dfrac{dy}{dx}$. **03**

(b) Solve $y'' + 2y' + 4y = 2x^2 + 3e^{-x}$ by undetermined coefficient method. **04**

(c) Find the power series solution of $y'' + xy' + x^2 y = 0$ about $x = 0$. **07**

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